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FLUCTUATIONS IN THE RATE OF FLOW DURING FILTRATION OF POLYMER SOLUTIONS

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UDC 542.67:541.6

This article demonstrates the possibility of a loss of stability in steady-state regimes involved in the filtration of polymer solutions, and we present also the construction of a mathematical model on the basis of whose analysis we have ascertained the unique features of self-oscillations and stochastic oscillations which arise in the region of instability.

Nonlinear effects in the filtration of non-Newtonian media may lead to a loss of stability in the steady filtration regime [1-4]. We observed such phenomena in a number of laboratory experiments in which we studied the filtration of polyacrylamide (PAA) solutions through a column filled with quartz sand. The permeability of the porous medium with respect to air amounted to $3.1 \cdot 10^{-12}$ m^2. During the course of the experiment the pressures at the inlet and outlet of the column were maintained at constant levels and the flow rate of the fluid being filtered was measured over a rather prolonged period of time. The experiments demonstrated that with small pressure differences a steady flow rate is established. But when some critical pressure difference Δp_* is attained (dependent on the PAA concentration in the solution) the steady filtration regimes lose stability, and we observe unattenuated fluctuations in the flow rate. As an example, Fig. 1 shows the flow rate for PAA with a concentration of 0.075% as a function of time for the case in which $\Delta p = 0.6$ MPa.

The fluctuations in the flow rate are irregular in nature. The level of irregularity (chaos) can be evaluated on the basis of the Hausdorf scale for the curve $Q = Q(t)$. The quantity D is determined [5, 6] during the process of measuring the length l on the curve $Q = Q(t)$ by means of dividers with an opening η. The measurements are started from the origin P_0. Describing a circle of radius η with the center at P_0, we mark the point P_1 at which the curve initially moves out of the circle. The second point P_2 is obtained when the center of the circle is shifted to the point P_1, etc. If $l(\eta)$ is used to denote the length of the resulting broken line $P_0P_1P_2...$, approximately describing the curve, the length of the curve will be [7, 8].

As demonstrated by direct measurement, for the experimental curves $Q = Q(t)$ with not overly small η, $l(\eta) \sim \eta^{-\gamma}$. Consequently, the graph of the functions $Q = Q(t)$ can be assumed to be fractal curves having the dimension $D = \gamma + 1$. It is natural to assume that the larger the dimension of the experimental curve, the less orderly the process whose image is represented by this curve. Thus, for the curve in Fig. 1 we have $D = 1.40$. We should take note of the fact that after establishment of the chaotic filtration regime any further increase in the pressure difference will not lead to an increase, but rather to a decrease in the Hausdorf dimension for the curves $Q = Q(t)$, which gives evidence of the more orderly progress of the filtration process in the case of larger values of Δp.

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Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 59, No. 2, pp. 211-215, August, 1990. Original article submitted July 28, 1989.

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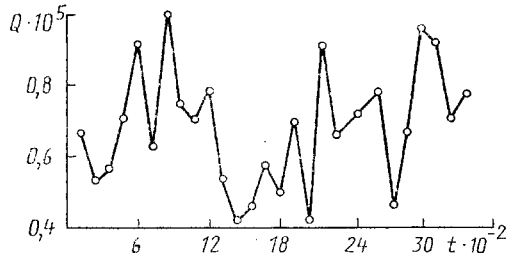

Fig. 1. The rate of flow for the PAA solution as a function of time. Q, m^3/sec; t, sec.

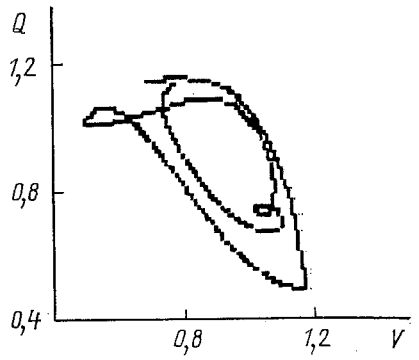

Fig. 2. Limit cycle corresponding to doubled period for B = 1.53, in coordinates Q = v(t), V = v(t − τ).

Let us examine the model which allows us to explain the appearance of fluctuations in the flow rate in the filtration of polymer solutions. For the sake of simplicity we will resort to the identification approach according to which this system is treated as a transmission unit to whose input a constant quantity, i.e., the pressure difference Δp, is transmitted, while at the output we observe a change in the filtration rate over time v(t) [9]. As was demonstrated in [10, 11], the nonsteady filtration process can be described within the framework of this approach by a concentrated model of the form

$$\lambda \frac{dv(t)}{d(t)} + v(t) = c\frac{\Delta p}{L}. \tag{1}$$

As a consequence of the appearance of polymer solutions exhibiting non-Newtonian properties, the filtration factor c depends on the rate of filtration. Since the structural transformation in polymer systems is characterized by retardation phenomena, this relationship can be represented in the form $c\,|_{t=t_1} = c(v(t_1 - T))$.

Let us make the form of the function c(v) more specific. With this in mind, we should note that it follows from (1) that in the steady regime we have v = c(v)Δp/L. With low rates of filtration for the polymer solutions we note the existence of an initial pressure gradient $\Delta p_0/L$, so that the function c(v) must satisfy the condition $\lim_{v\to 0} \frac{v}{c(v)} = \frac{\Delta p_0}{L}$.

Following [10], we will assume that with rapid motion of the polymer clumps they congeal at the narrowest points within the pores. This leads to a reduction in the rate of filtration with large values of v. For the sake of definiteness we will present the function c(v) with the above-noted properties in the following form [12]:

$$c(v) = \frac{L}{\Delta p_0}\frac{v}{1 + Mv^N}, \quad N > 1. \tag{2}$$

Changing over to dimensionless variables $t \to t/\lambda$, $\tau = T/\lambda$, $v \to v/v_0$, $v_0 = M^{-1/N}$, and $B = \Delta p/\Delta p_0$, from (1) and (2) we obtain that

$$\frac{dv(t)}{dt} + v(t) = \frac{Bv(t-\tau)}{1 + v^N(t-\tau)}. \tag{3}$$

As demonstrated by analysis [12, 13], Eq. (3) exhibits an equilibrium point v = 0 such that with B > 1 (i.e., with $\Delta p > \Delta p_0$) loses stability. In this case, the system assumes a new equilibrium position $v = v_1 = (B - 1)^{1/N}$. Any further

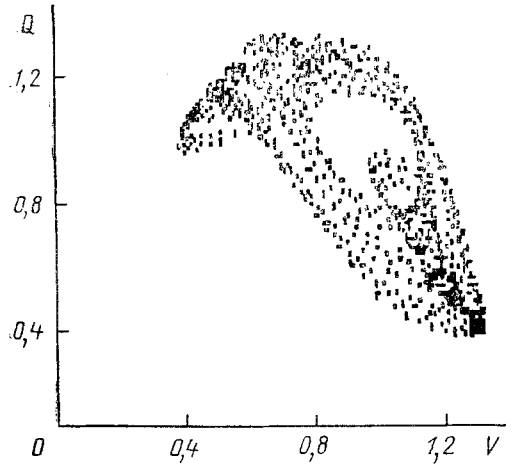

Fig. 3

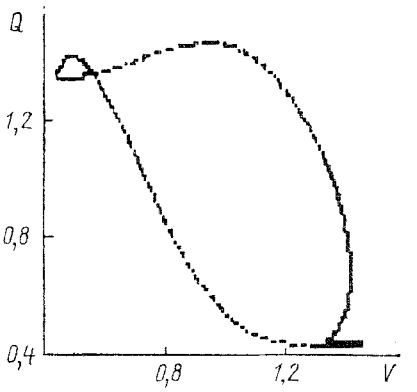

Fig. 4

Fig. 3. Strange attractor corresponding to B = 1.74, in coordinates Q = v(t), V = v(t – τ).

Fig. 4. Limit cycle corresponding to B = 2.22, in coordinates Q = v(t), V = v(t – τ).

increase in the parameter B leads to a situation at the critical point $B = B_0$ such that the steady regime of filtration at a rate $v = v_1$ also becomes unstable. Periodic and stochastic oscillations arise within the system. The value of B_0 can be derived by the method of D divisions [14] and it is equal to $B_0 = N/(N - 1 + \sec \varepsilon)$, while the quantity ε is determined from the equation $\tau = -\varepsilon \operatorname{cotan} \varepsilon$, $\pi/2 < \varepsilon < \pi$.

Let us present a number of quantitative estimates. The duration λ of the piezoconductivity is on the order of l^2/κ [10, 11]. We determines this quantity on the basis of pressure-restoration curves read in advance from a column. We found that $\lambda \sim 0.5$-1 min. The delay time T depends on the polymer concentration and ranges from 5-10 min to 1-2 h [10]. For the polymer solutions used in our experiments we can assume that T ~ 5 min. Assuming that $\lambda \sim 1$ min, we obtained the estimate $\tau \sim 5$. It is easy to calculate that with such a value of τ, $B_0 \approx N/(N - 2.1)$. We do not have the necessary data at hand to obtain estimates of the magnitude of N, but the very fact that the loss of stability in steady filtration for polymer solutions was experimentally observed serves as an indirect proof of the fact that the quantity N is sufficiently large (for $\tau = 5$ we have at least $N > 2.1$).

Choosing the function c(v) in the form of (2) assumes that with an increase in the filtration rate the quantity c tends to vanish. A more general situation arises when the coefficient of filtration in the case of large v tends to some asymptotic value different from zero. We therefore conducted our calculations with a function c(v) of the form

$$c(v) = v\,[\exp(-v^N) + G/(Av + 1)],$$

for which $\lim_{v \to \infty} c(v) = G/A > 0$.

We used an exponent in this expression, rather than an exponential function of the form $1/(1 + v^N)$ in order to test the stability of the derived results relative to changing the means by which we parametrized the function c(v).

Calculations showed that the effects produced by the appearance of periodic and stochastic self-oscillations are present in this case as well. We will present here the results obtained for A = 10, G = 2, N = 5, and $\tau = 5$.

Initially, the increase in the parameter B leads through the period-doubling bifurcation circuit to the points $B_1 \approx 1.20$, $B_2 \approx 1.46$, $B_3 \approx 1.60$, ...; this sets up the chaotic regime. Any further increase in B causes the motion in the system to become ordered. Limit cycles arise and their periods, as B continues to grow, successively are reduced to half, i.e., inverse Feigenbaum bifurcations appear [15]. Finally, with some sufficiently large value of B a stationary state is once again established.

To illustrate the described scenario, Figs. 2-4 show the attractors which correspond to a motion with period 2 (B = 1.53), the chaotic regime (B = 1.74), and motion with period 1 (B = 2.22).

The reverse transition from chaos to the steady state, observed in our study of the proposed model, might serve as an explanation of the fact that the increase in the pressure difference leads to a reduction in the irregularity of the functions Q = Q(t).

We have thus demonstrated that the fluctuations in the rate of filtration flow in the case of polymer solutions can be explained by three factors: the presence of an initial pressure gradient, the flow "blockage" effect of polymer molecule clusters, and the existence of some delay time in the processes of structural transformation in polymer systems. The qualitative conclusions drawn from an analysis of the proposed model are in agreement with the experimentally derived results.

These results may find extensive application in the development of strategies to control the pumping of polymer solutions through oil-bearing strata. They make it possible, in particular, to specify pumping regimes which exclude the possibility of chaotic fluctuations.

NOTATION

p, pressure; Δp, pressure difference; Q, fluid flow rate; D, Hausdorf dimension; t, time; v, filtration rate; c, filtration factor; L, length of experimental column; λ, duration of piezoconductivity; κ, coefficient of piezoconductivity; T, delay time; γ, A, G, N, positive-definite constants; B, bifurcation parameter.

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